Chapter 2.3 Practice: Conditional Probability

Solutions

1. A hat contains six slips of paper with the numbers 1 through 6 written on them. Two slips of paper are drawn from the hat without replacement and the sum of the numbers is computed.

a. What is the probability that the sum of the numbers is exactly 10?

```
hat_sample_10 <- replicate(10000,{
  sum_sample <- sum(sample(1:6,2))
  sum_sample==10
})

mean(hat_sample_10)
```

```
## [1] 0.0713
```

b. What is the probability that the sum of the numbers is at least 10?

```
hat_sample_at_least_10<-replicate(10000,{
  sum_sample<-sum(sample(1:6,2))
  sum_sample>9
})

mean(hat_sample_at_least_10)
```

```
## [1] 0.1355
```

c. What is the probability that the sum of the numbers is exactly 10, given that it is at least 10?

```
mean(hat_sample_10)/mean(hat_sample_at_least_10)
```

```
## [1] 0.5261993
```

2. Roll two dice, one white and one red. Consider these events:

- A: The sum is 7
- B: The white die is odd
- C: The red die has a larger number showing than the white
- D: The dice match (doubles)

a. Which pair(s) of events are disjoint?

- The events A and D are disjoint because 7 is an odd number, and 2*k where k is the number showing on the die will always be even, and a number can't be both even and odd at the same time.
- C and D are disjoint because if the faces of the red and white die match, then one can't be larger than the other.

b. Which pair(s) are independent?

Events B and D are independent because the probability of one die being odd does not affect or change the probability that the two dice match (and vice versa).

A and B are independent because for $P(A|B) = P(B)$

c. Which pair(s) are neither disjoint nor independent?

The events A and C, B and C. For all of these pairs, knowing that one event has happened *changes* the probability that the other will occur. Also, these pairs are not disjoint because there are cases where both events can occur simultaneously.

3. Let A and B be events. Show that $P(A \cup B|B) = 1$.

$P(A \cup B|B) = \frac{P((A \cup B) \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$

4. Suppose a die is tossed three times. Let A be the event *the first toss is a 5*. Let B be the event *the first toss is the largest number rolled* (the largest can be a tie). Determine via simulation whether A or B are independent.

If A and B were independent, then $P(A \cap B) = P(A)P(B)$. So let's simulate each case.

- Event A

```r
event_a <- replicate(10000,{
  toss_3  <- sample(1:6,3,replace=TRUE)
  event_a <- toss_3[1]==5
})
mean(event_a)
```

```
## [1] 0.1625
```

- Event B

```r
event_b <- replicate(10000,{
  toss_3  <- sample(1:6,3,replace=TRUE)
  event_b <- max(toss_3)==toss_3[1]
})
mean(event_b)
```

```
## [1] 0.4195
```

- $P(A \cap B)$

```r
ab_intersect<-replicate(10000,{
  toss_3 <- sample(1:6,3,replace=TRUE)
  ab_intersect <- (toss_3[1]==5) & (max(toss_3)==toss_3[1])
})

mean(ab_intersect)
```

```
## [1] 0.1182
```

- $P(A) * P(B)$

```r
mean(event_a)*mean(event_b)
```

```
## [1] 0.06816875
```

Since $P(A) * P(B)$ isn't close to $P(A \cap B)$, it doesn't appear that A and B are independent.

5. Bob Ross was a painter with a PBS television show "The Joy of Painting' ' that ran for 11 years. One report showed that 91% of Bob's paintings contain a happy little tree, 85% contain two or more happy trees. What is the probability that he painted a second happy tree to be a friend for the first happy tree?

$P(2trees|1tree) = \frac{P(2treesand1tree)}{P(1tree)} = 0.85/0.91 = 0.934$