Ch 3.3 Practice: Functions of Random variables, and the Variance

Solutions

1. Let X be a discrete random variable with pmf given by

P(X=0)=1/4, P(X=1)=1/2, P(X=2)=1/8, P(X=3)=1/8

a. Find the pmf of $Y = X - 1$.

```
X <- 0:3
Y <- X-1
prob_Y <- c(0.25,0.5,.125,.125)
kable(rbind(Y, prob_Y))
```

Y	-1.00	0.0	1.000	2.000
prob_Y	0.25	0.5	0.125	0.125

b. Find the theoretical pmf of $V = (X - 1)^2$.

Note that there are two ways to get V = 1, so the pmf needs to be simplified.

```
(V <- (X-1)^2)
```

```
## [1] 1 0 1 4
```

```
(V <- unique(V))
```

```
## [1] 1 0 4
```

```
prob_V <- c(prob_Y[1]+prob_Y[3], prob_Y[2], prob_Y[4])
kable(rbind(V, prob_V))
```

V	1.000	0.0	4.000
prob_V	0.375	0.5	0.125

c. Calculate the theoretical values for $E(V)$ and $Var(V)$.

```
(mu <- sum(V*prob_V))
```

```
## [1] 0.875
```

```
(variance <- sum(V^2*prob_V) - mu^2)
```

```
## [1] 1.609375
```

d. Estimate $E(V)$ and $Var(V)$ using simulation.

```
samp.v <- sample(V, size=10000, prob=prob_V, replace=TRUE)
mean(samp.v)
```

```
## [1] 0.87
```

```
var(samp.v)
```

```
## [1] 1.609661
```

2. Let X and Y be random variables such that $E[X] = 2$ and $E[Y] = 3$. Answer the following questions and show your work using equations not just numbers.

a. Find $E[4X + 5Y]$.

```
(E_4X_5Y <- 4*2 + 5*3)
```

```
## [1] 23
```

b. Find $E[4X - 5Y + 2]$.

```
(E_4X_5Y_2 <- 4*2 - 5*3 + 2)
```

```
## [1] -5
```

3. Roll two ordinary dice and let X be their sum.

a. Compute the pmf for X. Hint: Use simulation to calculate the pmf of the sum of two dice using the `proportions()` function, then save the result of that pmf as a new object `p.x`

```
roll <- replicate(10000,{
  sum(sample(1:6,2,replace=TRUE))
})

X <- 2:12
prob_X <- proportions(table(roll))
kable(rbind(X, prob_X), digits=3)
```

	2	3	4	5	6	7	8	9	10	11	12
X	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000	11.000	12.000
prob_X	0.026	0.055	0.085	0.114	0.139	0.164	0.135	0.116	0.083	0.057	0.025

b. Compute the theoretical mean and standard deviation of X.

```
(E_X <- sum(X*prob_X))
```

```
## [1] 6.9992
```

```
(Var_X <- sum(X^2*prob_X)-E_X^2)
```

```
## [1] 5.777799
```

```
sqrt(Var_X)
```

```
## [1] 2.403705
```

c. Confirm your result for the mean and standard deviation using simulation.

```
mean(roll)
```

```
## [1] 6.9992
```

```
sd(roll)
```

```
## [1] 2.403826
```

4. Let X be a random variable with mean μ and standard deviation σ. Use the properties of the variance to find the mean and standard deviation of $\frac{X-\mu}{\sigma}$.

$$\begin{aligned} E(X) &= E\left(\frac{X-\mu}{\sigma}\right) \\ &= \frac{1}{\sigma}E(X-\mu) \\ &= \frac{1}{\sigma}(E(X)-\mu) \\ &= \frac{1}{\sigma}(\mu-\mu) \\ &= 0 \end{aligned}$$

$$\begin{aligned} Var(X) &= Var\left(\frac{X-\mu}{\sigma}\right) \\ &= \frac{1}{\sigma^2}Var(X-\mu) \\ &= \frac{1}{\sigma^2}[Var(X)+Var(\mu)] \\ &= \frac{1}{\sigma^2}Var(X) \\ &= \frac{1}{\sigma^2}\sigma^2 \\ &= 1 \end{aligned}$$