Ch 4.1 Practice: PDFs and CDFs

Solutions

1. Let X be a random variable with pdf given by $f(X) = 2x$ for $0 \leq x \leq 1$ and 0 otherwise.

a. Find $P(X \geq 1/2)$

```
integrand<-function(x) {2*x}
integrate(integrand,lower=1/2,upper=1)
```

```
## 0.75 with absolute error < 8.3e-15
```

b. Find $P(X \geq 1/2|X \geq 1/4)$.

$P(X \geq 1/2|X \geq 1/4) = \frac{P(X \geq 1/2 \cap X \geq 1/4)}{P(X \geq 1/4)} = \frac{P(X \geq 1/2)}{P(X \geq 1/4)}$

```
integrate(integrand,lower=.5,upper=1)
```

```
## 0.75 with absolute error < 8.3e-15
```

```
integrate(integrand,lower=.25,upper=1)
```

```
## 0.9375 with absolute error < 1e-14
```

```
.75/.9375
```

```
## [1] 0.8
```

2. Let X be a random variable with pdf given below. Find the constant c such that this is a valid pdf.

$$f(x) = \begin{cases} cx^2 & \text{for } 0 \leq x < 1 \\ c(2-x)^2 & \text{for } 1 \leq x \leq 2 \\ 0 & otherwise. \end{cases}$$

Find the value of c such that

$$\int_0^1 cx^2 dx + \int_1^2 c(2-x)^2 dx = 1 \longrightarrow c\left(\int_0^1 x^2 dx + \int_1^2 (2-x)^2 dx\right) = 1$$

First find out what the current density is for each part of this piecewise function.

```
part1 <- function(x) {x^2}
(int1 <- integrate(part1,lower=0,upper=1))
```

```
## 0.3333333 with absolute error < 3.7e-15
```

```
part2 <- function (x) {(2-x)^2}
(int2 <- integrate(part2,lower=1,upper=2))
```

```
## 0.3333333 with absolute error < 3.7e-15
```

Plugging these numbers into the equation above:

$$c(.333 + .333) = 1 \rightarrow c = 1/.67$$

3. Suppose that the p.d.f. of a random variable X is as follows:

$$f(x) = \begin{cases} \frac{1}{8}x & for\ 0 \leq x \leq 4 \\ 0 & otherwise. \end{cases}$$

a. Find the value of t such that $\mathbf{P}(X \leq t) = 1/4$.

$\int_0^t \frac{1}{8}xdx = \frac{1}{4} \rightarrow t = 2$

b. Find the value of t such that $\mathbf{P}(X \geq t) = 1/2$.

$\int_t^4 \frac{1}{8}xdx = \frac{1}{2} \rightarrow t = \sqrt{8}$

4. Suppose $f_Y(y) = 4y^3$, $0 \leq y \leq 1$. Find $P(0 \leq Y \leq \frac{1}{2})$

```
integrand<-function(x) {4*x^3}
integrate(integrand,lower=0,upper=.5)
```

```
## 0.0625 with absolute error < 6.9e-16
```

5. Let $f(y) = \frac{3}{2}y^2$ for $-1 \leq y \leq 1$. Find $P(|Y - \frac{1}{2}| < \frac{1}{4})$.

```
integrand<-function(y) {1.5*y^{2}}
integrate(integrand,lower=0.25,upper=.75)
```

```
## 0.203125 with absolute error < 2.3e-15
```

6. The cdf for a random variable Y is defined by $F(y) = 0$ for $y < 0$; $F(y) = 4y^3 - 3y^4$ for $0 \leq y \leq 1$; and $F(y) = 1$ for $y > 1$. Find $P(\frac{1}{4} \leq Y \leq \frac{3}{4})$.

```r
cdf <- function(y){4*y^3 - 3*y^4}
cdf(.75) - cdf(.25)
```

```
## [1] 0.6875
```