Ch 4.2 & 4.3 Practice: Expected value & Variance of a continuous random variable

YOUR NAME HERE

1. Calculate $E(Y)$ for the following pdfs:

a. $f(y) = 3(1-y)^2$, $0 \le y \le 1$

$$E(Y) = \int_0^1 y * 3(1-y)^2 dy$$

```
integrand <- function(y){3*y*(1-y)^2}
integrate(integrand,lower=0,upper=1)
```

```
## 0.25 with absolute error < 2.8e-15
```

b. $f(y) = 4ye^{-2y}$, $y \ge 0$

$$E(Y) = \int_0^\infty y * 4ye^{-2y} dy$$

```
integrand <- function(y){y^2*4*exp(-2*y)}
integrate(integrand,lower=0,upper=Inf)
```

```
## 1 with absolute error < 7.9e-05
```

2. If Y has pdf $f(y) = 2y$ for $0 < y < 1$. Then $E(Y) = \frac{2}{3}$. Define the random variable W to be squared deviation of Y from its mean, that is, $W = (Y - \frac{2}{3})^2$. Find $E(W)$.

$$E(W) = \int w(y) * f(y)dy = (y - \frac{2}{3})^2 * 2y \; dy$$

```
integrand <- function(y){2*y*(y-2/3)^2}
integrate(integrand,lower=0,upper=1)
```

```
## 0.05555556 with absolute error < 6.2e-16
```

3. A box is to be constructed so that its height is five inches and its base is Y inches by Y inches, where Y is a random variable described by the pdf, $f(y) = 6y(1-y)$, $0 < y < 1$. Find the expected value of the volume of the box.

The formula for the volume of the box is $5y^2$. So,

$$E(Volume) = \int_0^1 5y^2 * 6y(1-y)\ dy$$

```
integrand <- function(y){5*y^2*6*y*(1-y)}
integrate(integrand,lower=0,upper=1)
```

```
## 1.5 with absolute error < 1.7e-14
```

4. Let X be a random variable with pdf $f(x) = 3(1-x)^2$ when $0 \leq x \leq 1$ and $f(x) = 0$ otherwise.

```
pdf.for.q4 <- function(x){3*(1-x)^2}
```

a. Verify that f is a valid pdf.

- Condition 1: The integral of the pdf over the domain of support equals 1.

```
integrate(pdf.for.q4,lower=0,upper=1)
```

```
## 1 with absolute error < 1.1e-14
```

- Condition 2: $f(x)$ is nonnegative across the domain of support.

```
x <- seq(0, 1, by=.01)
f.x <- pdf.for.q4(x)
min(f.x)
```

```
## [1] 0
```

Both condition 1 and 2 are met, so it is a valid pdf.

b. Find the mean and variance of X.

```
integrand <- function(x){3*x*(1-x)^2}
q4.e_x <- integrate(integrand,lower=0,upper=1)
(E_X <- q4.e_x$value)
```

```
## [1] 0.25
```

```
integrand<-function(x) {3*x^2*(1-x)^2}
q4.e_xsq <- integrate(integrand,lower=0,upper=1)
(E_X_sq <- q4.e_xsq$value)
```

```
## [1] 0.1
```

```
(Var_X <- E_X_sq - E_X^2)
```

```
## [1] 0.0375
```

c. Find $P(X \leq 1/2)$.

```
integrate(pdf.for.q4,lower=0,upper=.5)
```

```
## 0.875 with absolute error < 9.7e-15
```

d. Find $P(X \leq 1/2 | X \geq 1/4)$.

$$P(X \leq 1/2 | X \geq 1/4) = \frac{P(X \leq 1/2 \cap X \geq 1/4)}{P(X \geq 1/4)} = \frac{P(1/4 \leq X \leq 1/2)}{P(X \geq 1/4)}$$

```r
(x_lt.5_and_gt.25 <- integrate(pdf.for.q4,lower=.25,upper=.5))
```

```
## 0.296875 with absolute error < 3.3e-15
```

```r
(x_gt.25 <- integrate(pdf.for.q4,lower=.25,upper=1))
```

```
## 0.421875 with absolute error < 4.7e-15
```

```r
x_lt.5_and_gt.25$value / x_gt.25$value
```

```
## [1] 0.7037037
```

5. Let $f(y) = 3(1-y)^2$ for $0 < y < 1$. Find the variance of W where $W = -5Y + 12$.

$$Var(W) = Var(-5Y + 12) = 25Var(Y)$$

```
integrand<-function(y) {y*3*(1-y)^2}
(E_Y <- integrate(integrand,lower=0,upper=1))
```

```
## 0.25 with absolute error < 2.8e-15
```

```
integrand<-function(y) {y^2*3*(1-y)^2}
(E_Ysq <- integrate(integrand,lower=0,upper=1))
```

```
## 0.1 with absolute error < 1.1e-15
```

```
(Var_Y <- 25*(E_Ysq$value - (E_Y$value)^2))
```

```
## [1] 0.9375
```

Alternative approach

$$Var(W) = E(W^2) - E(W)^2$$

```
integrand<-function(y) {(-5*y+12)*3*(1-y)^2}
(E_W <- integrate(integrand,lower=0,upper=1))
```

```
## 10.75 with absolute error < 1.2e-13
```

```
integrand<-function(y) {(-5*y+12)^2*3*(1-y)^2}
(E_Wsq <- integrate(integrand,lower=0,upper=1))
```

```
## 116.5 with absolute error < 1.3e-12
```

```
(Var_W <- E_Wsq$value - (E_W$value)^2)
```

```
## [1] 0.9375
```