Ch 4.4 Practice: Normal random variables

SOLUTIONS

Make sure you read the question carefully. Sometimes these questions require a couple of steps, and some questions ask you to use other distributions we've talked about.

1. Suppose that scores on an exam are normally distributed with mean 80 and standard deviation 5.

Let X be the exam score. $X \sim N(80, 25)$.

a. What is the probability that a student scores higher than 85 percent on the exam?

Find: $P(X \geq 85)$

```
mu <- 80
sigma <- 5
(p <- 1-pnorm(85,mu,sigma))
```

```
## [1] 0.1586553
```

The probability that a student scores higher than 85 percent on the exam is 0.159

b. Assume that exam scores are independent and that 10 students take the exam. What is the probability that 4 or more students score 85 percent or higher on the exam.

Let Y be the number of students scoring over 85% on the exam. $Y \sim Binomial(10,$ `r round(p,3)‘). Find $P(X \geq 4) = 1 - P(X \leq 3)$

```
(b4 <- 1-pbinom(3,10,p))
```

```
## [1] 0.05974504
```

The probability that 4 or more students score 85 percent or higher on the exam is 0.06

2. Climbing rope will break if pulled hard enough. Experiments show that 10.5 mm Dynamic rope has a mean breaking point of 5036 lbs with a standard deviation of 122 lbs. Assume breaking points of rope are normally distributed.

Let X be the breaking point in lbs of a rope. $X \sim N(5036, 14884)$.

a. What proportion of ropes will break with at least 5000 lbs of load?

Find: $P(X \geq 5000)$.

```
mu <- 5036
sigma <- 122

(rope <- 1 - pnorm(5000,mu,sigma))
```

```
## [1] 0.6160344
```

5000 lbs of load will break 61.6% of ropes.

b. At what load will 95% of all ropes break?

Find t such that $P(X < t) = .95$.

```
(heavy <- qnorm(.95,mu,sigma))
```

```
## [1] 5236.672
```

95% of all ropes break at 5236.67 lbs of force.

3. Econo-Tire is planning on advertising campaign for its newest product, an inexpensive radial. Preliminary road tests conducted by the firm's quality-control department have suggested that the lifetimes of a set of these tires will be normally distributed with an average of 30,000 miles and a standard deviation of 5000 miles.

a. The marketing division would like to run a commercial that makes the claim that at least nine out of ten drivers will get at least 25,000 miles for a set of Econo Tires. Based on the road test data, is the company justified in making that assertion?

Let X be the lifetime of tires. $X \sim N(30000, 5000^2)$. Find $P(X > 25k)$

```
mu <- 30000
sigma <- 5000

(p.25k <- 1-pnorm(25000,mu,sigma))
```

```
## [1] 0.8413447
```

The probability of a set of tires lasting past 25k is 0.84.

Now let Y be the number of drivers who get at least 25k out of their tires. $Y \sim Binomial(10, 0.84)$. We want to find $P(X \geq 9) = 1 - P(X \leq 8)$.

```
1-pbinom(8,10,p.25k)
```

```
## [1] 0.5128569
```

b. What mileage corresponds to the 25th percentile for tire lifetime?

```
qnorm(.25,mu,sigma)
```

```
## [1] 26627.55
```

4. Speeding on I-5: The distribution of passenger vehicle speeds traveling on the Interstate 5 Freeway (I-5) in California is nearly normal with a mean of 72.6 miles/hour and a standard deviation of 4.78 miles/hour.

a. Define your random variable.

Let X be the speed of a passenger vehicle traveling on I5.

b. Write distributional Notation

$$X \sim Normal(72.6, 4.78^2)$$

State what you are trying to find in math notation, and then answer the following questions using `pnorm` and using simulation.

c. What percent of passenger vehicles travel slower than 80 miles/hour?

Find: $P(X < 80)$

```
pnorm(80, 72.6, 4.78)
```

```
## [1] 0.939203
```

```
speed <- rnorm(10000, 72.6, 4.78)
mean(speed < 80)
```

```
## [1] 0.9403
```

d. What percent of passenger vehicles travel between 60 and 80 miles/hour?

Find: $P(60 < X < 80)$

```
pnorm(80, 72.6, 4.78) - pnorm(60, 72.6, 4.78)
```

```
## [1] 0.9350083
```

```
mean(speed < 80 & speed > 60)
```

```
## [1] 0.9366
```

e. The speed limit on this stretch of the I-5 is 70 miles/hour. Approximate what percentage of the passenger vehicles travel above the speed limit on this stretch of the I-5.

Find: $P(X > 70)$

```
1 - pnorm(70, 72.6, 4.78)
```

```
## [1] 0.7067562
```

```
mean(speed > 70)
```

```
## [1] 0.6985
```

State what you are trying to find in math notation, and then answer the following questions using R functions only (no simulation)

e. How fast do the fastest 5% of passenger vehicles travel?

Find: $P(X >?) = .05$

```
qnorm(.95, 72.6, 4.78)
```

```
## [1] 80.4624
```

5. Triathlon times

In triathlons, it is common for racers to be placed into age and gender groups. Friends Leo and Mary both completed the Hermosa Beach Triathlon, where Leo competed in the Men, Ages 30 - 34 group while Mary competed in the Women, Ages 25 - 29 group. Leo completed the race in 1:22:28 (4948 seconds), while Mary completed the race in 1:31:53 (5513 seconds). Obviously Leo finished faster, but they are curious about how they did within their respective groups. Can you help them? Here is some information on the performance of their groups:

- The finishing times of the Men, Ages 30 - 34 group has a mean of 4313 seconds with a standard deviation of 583 seconds.
- The finishing times of the Women, Ages 25 - 29 group has a mean of 5261 seconds with a standard deviation of 807 seconds.
- The distributions of finishing times for both groups are approximately Normal.

Remember: a better performance corresponds to a faster finish.

a. Write down the distributional notation for the race times for males and females separately.

Let X_m be the finish time for males, and X_f be the finish times for females.

$$X_m \sim N(4313, 339889) \qquad X_f \sim N(5261, 651249)$$

b. What percent of the triathletes did Leo finish faster than in his group?

We want to know what % of athletes came in after Leo, which means at a higher time.

Find $P(X_m > 4948)$

```
1-pnorm(4948, 4313, 583)
```

```
## [1] 0.1380342
```

Leo finished faster than 13.8% than others in his group.

c. What percent of the triathletes did Mary finish faster than in her group?

Find $P(X_f > 5513)$

```
1-pnorm(5513, 5261, 807)
```

```
## [1] 0.3774186
```

Mary finished faster than 37.7% than others in her group.

d. Did Leo or Mary rank better in their respective groups? Explain your reasoning.

Mary ranked better compared to Leo, because she finished in a higher percentile within her group.