Section 4.5 Practice: Uniform and Exponential random variables

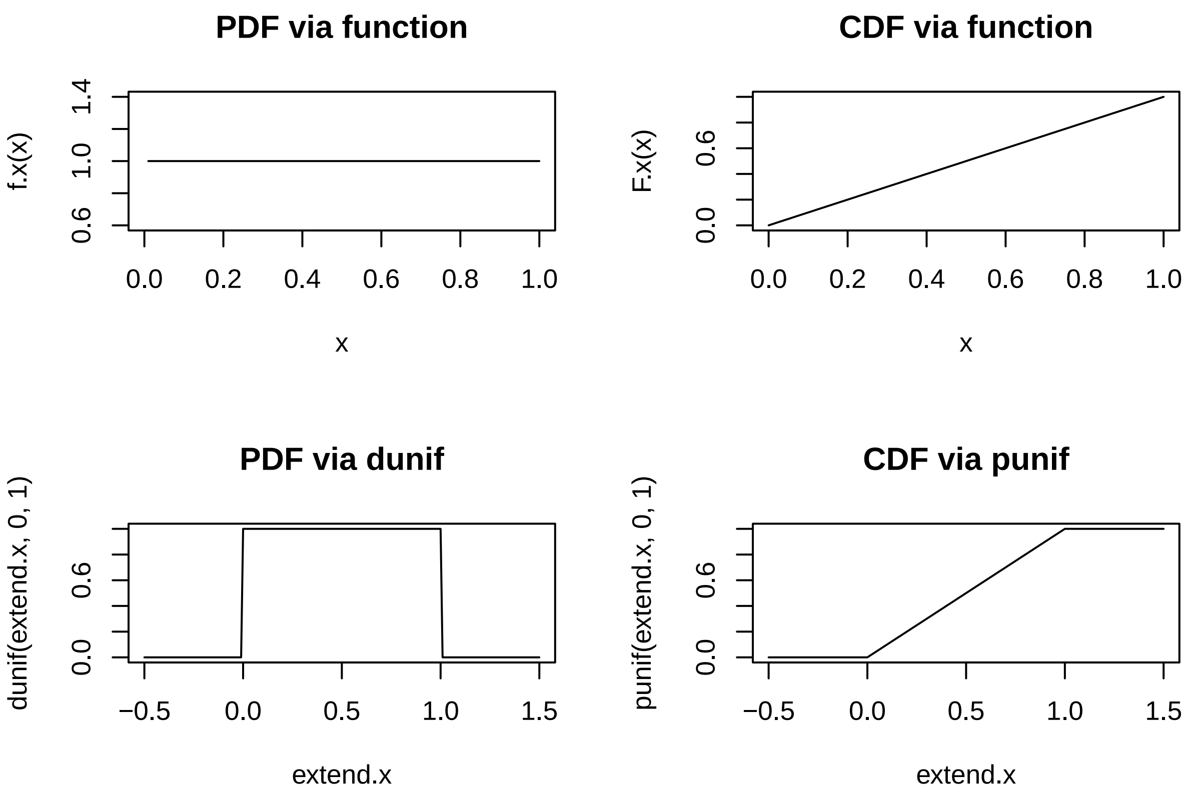
SOLUTIONS

1. Plot the pdf and cdf of a uniform random variable on the interval [0,1].

Hint: You may need to refer to ch1 notes to create these plots

```
x <- seq(0, 1, by=.01)
f.x <- function(x){x/(x*(1-0))}
F.x <- function(x){x/(1-0)}
extend.x <- seq(-.5, 1.5, by=.01)

par(mfrow=c(2,2))
plot(x, f.x(x), type = 'l', main = "PDF via function")
plot(x, F.x(x), type = 'l', main = "CDF via function")
plot(extend.x, dunif(extend.x, 0, 1), type = 'l', main = "PDF via dunif")
plot(extend.x, punif(extend.x, 0, 1), type = 'l', main = "CDF via punif")
```



Either set of plots is okay.

2. For each of the following descriptions of a random variable, indicate whether it can best be modeled by binomial, geometric, uniform, exponential or normal. Also, answer the associated question.

a. Suppose a stop light has a red light that lasts for 60 seconds a green light that lasts for 30 seconds and a yellow light that lasts for 5 seconds. When you first observe the stop light, it is red. Let X denote the time until the light turns green. What type of rv would be used to model X? What is the mean?

Stop light colors change from red, to green, to yellow before going back to red. So if you first observe the stop light when it's red, the next color will be green.

- $X \sim Uniform(0, 60)$
- $E(X) = (60 - 0)/2 = 30$

b. Customers arrive at a teller's window at a uniform rate 5 per hour. Let X be the length in minutes of time that the teller has to wait until they see their first customer after starting their shift. What type of rv is X? What is its mean? Find the probability that the teller waits less than 10 minutes for their first customer.

- $X \sim Exp(5/60)$
- $E(X) = \frac{60}{5} = 12$ minutes
- $P(X < 10)$

```
pexp(10, 5/60)
```

```
## [1] 0.5654018
```

c. Let X be the recorded body temperature of a healthy adult in degrees Fahrenheit. What type of rv is X? Estimate its mean and standard deviation, based on your knowledge of body temperatures.

According to [WebMD] and [Wikipedia], the average body temperature for most people is 98.2 degrees Fahrenheit, but often range from 97 to 99. So I'm going to estimate the standard deviation to be a little over 1. We can assume a Normal distribution because having a fever or having low body temperature is not as common as having a temperature around the average.

- $X \sim Normal(98.6, 4)$

3. Let X be an exponential rv with $\lambda = 1/4$. Answer the following questions using both theoretical and simulation methods.

```
x <- rexp(10000, .25) # generate sample
```

a. What is the mean of X.

- Theoretical

```
(E_x <- 1/.25)
```

```
## [1] 4
```

- Simulation

```
mean(x)
```

```
## [1] 4.01548
```

b. Find the probability of $P(.25 \leq X \leq 1)$ using both formulas and simulation.

- Theoretical via Integration

```
pdf.x <- function(x){0.25*exp(-0.25*x)}
integrate(pdf.x, lower=.25, upper=1)
```

```
## 0.1606123 with absolute error < 1.8e-15
```

- Theoretical via R commands

```
pexp(1, .25) - pexp(.25, .25)
```

```
## [1] 0.1606123
```

- Simulation

```
mean(x > .25 & x < 1)
```

```
## [1] 0.1609
```

4. The minimum of two independent exponential random variables with mean 2 is an exponential random variable. Use simulation to determine what the mean value is for the distribution (of the minimum of two independent exponential r.v. with mean 2).

```
v1 <- rexp(10000, 1/2)
v2 <- rexp(10000, 1/2)
v <- pmin(v1, v2)
mean(v)
```

```
## [1] 1.013035
```

5. Let X be an exponential random variable with rate λ. If a and b are positive numbers, then $P(X > a + b|X > b) = P(X > a)$. Explain how this equation demonstrates the *memoryless property* of the exponential.

The left hand side of the equation is the probability that we wait a units longer, given that we have already waited b units. The right hand side is the probability that we wait a units, from the beginning. Because these two probabilities are the same, it means that waiting b units has gotten us no closer to the occurrence of the event. The Poisson process has no memory that you have "already waited b units.

6. The number of days ahead travelers purchase their airline tickets can be modeled by an exponential distribution with the *average amount of time* equal to 15 days.

a. Define the random variable in a sentence, and distributional notation.

Let X be the number of days ahead travelers purchase their airline tickets. $X \sim Exp\left(\frac{1}{15}\right)$

b. Find the probability that a traveler will purchase a ticket fewer than ten days in advance.

Find: $P(X < 10)$

```
pexp(10, 1/15)
```

```
## [1] 0.4865829
```

c. How many days do half of all travelers wait?

Find: $P(X < t) = .5$

```
qexp(.5, 1/15)
```

```
## [1] 10.39721
```

Half of all travelers wait 10 days.