MATH 350: Practice Exam 3

1. Let $f(y) = \frac{3}{14}(y^2+1)$ where $0 < y < 2$. Find $P(Y > 1|Y < 1.5)$

$$
\begin{aligned}
P(Y > 1|Y < 1.5) &= \frac{P(Y > 1 \cap Y < 1.5)}{P(Y < 1.5)} = \frac{P(1 < Y < 1.5)}{P(Y < 1.5)} \\
&= \int_1^{1.5} \frac{3}{14}(y^2+1) \div \int_0^{1.5} \frac{3}{14}(y^2+1)
\end{aligned}
$$

```
fn <- function(y){(3/14)*(y^{2}+1)}
p.btwn.1.and.1.5 <- integrate(fn, lower = 1, upper= 1.5)
p.lt.1.5 <- integrate(fn, lower = 0, upper= 1.5)
p.btwn.1.and.1.5$value / p.lt.1.5$value
```

```
## [1] 0.4920635
```

2. Suppose that a random variable X has a pdf $3x^2$ from $0 < x < 1$. Compute the cdf.

$$F(X) = \int_0^x 3t^2 \; dt = t^3 \Big|_0^x = x^3$$

3. Is the following function a valid pdf? $f(x) = \frac{1}{12}x + \frac{1}{4}$ where $2 < x < 4$. Explain your answer.

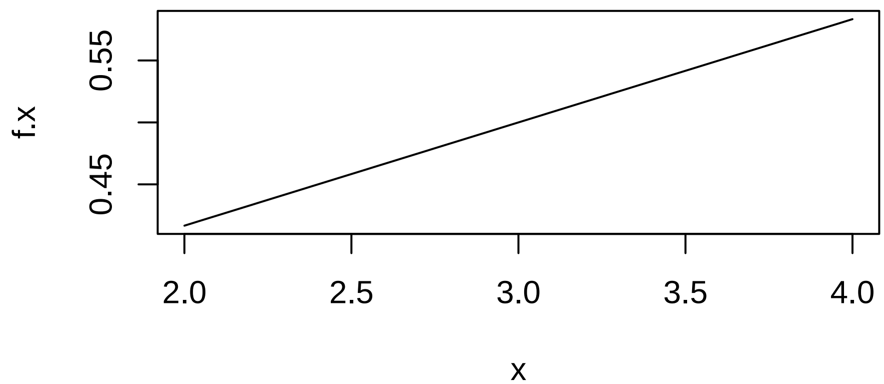
Check 1: Does $f(x)$ integrate to 1 over the domain of support?

```
pdf <- function(x){1/12*x + 1/4}
integrate(pdf, 2, 4)
```

```
## 1 with absolute error < 1.1e-14
```

Check 2: Are all $f(x) > 0$?

```
x <- seq(2, 4, length.out=100)
f.x <- pdf(x)
plot(x, f.x, type = 'l')
```



Yes. The function integrates to 1 over its domain of support. Since this is a linear function with a positive intercept and slope over the domain, it will always stay positive.

4. If Y has pdf $f(y) = 4y - 1$ where 0 < y < 1 and $Z = 2y^2$. Find $Var(Z)$.

$$\begin{aligned}
E(Z) &= \int_0^1 2y^2 * (4y-1)\ dy \\
&= \int_0^1 8y^3 - 2y^2\ dy \\
&= \frac{8}{4}y^3 - \frac{2}{3}y^3\Big|_0^1 \\
&= 2 - \frac{2}{3} = \frac{4}{3}
\end{aligned}$$

```
fun1 <- function(y){(2*y^2)*(4*y-1)}
(E_Z <- integrate(fun1, 0, 1))
```

```
## 1.333333 with absolute error < 1.5e-14
```

$$\begin{aligned}
E(Z^2) &= \int_0^1 (2y^2)^2 * (4y-1)\ dy \\
&= \int_0^1 16y^4 - 4y^4\ dy \\
&= \frac{16}{6}y^6 - \frac{4}{5}y^5\Big|_0^1 \\
&= \frac{80}{30} - \frac{24}{30} = \frac{28}{15}
\end{aligned}$$

```
fun2 <- function(y){(2*y^2)^2*(4*y-1)}
(E_Zsq <- integrate(fun2, 0, 1))
```

```
## 1.866667 with absolute error < 2.1e-14
```

$$Var(Z) = E(Z^2) - E(Z)^2 = \frac{28}{15} - \frac{16}{9} = 0.89$$

```
E_Zsq$value - E_Z$value^2
```

```
## [1] 0.08888889
```

5. Fueleconomy.gov, the official US government source for fuel economy information, allows users to share gas milage information on their vehicles. A large sample of data 2012 Toyota Prius drivers who entered data on this site showed that MPG was approximately normally distributed with an average of 53.3 MPG and a standard deviation of 5.2 MPG.

For part b-d, write what you are trying to find in math notation, write the R code for the theoretical answer, and if the question is asking you to find a probability also write the simulation code.

a. Define a random variable X and write its distributional notation.

Let X be the MPG of a 2012 Toyota Prius. $X \sim N(53.3, 5.2)$

```
mpg <- rnorm(10000, 53.3, 5.2)
```

b. The EPA claims that a typical 2012 Prius gets at least 50MPG. What is the probability that the EPA's estimate is accurate?

Find: $P(X > 50)$

```
1-pnorm(50, 53.3, 5.2)
```

```
## [1] 0.7371604
```

```
mean(mpg>50)
```

```
## [1] 0.7298
```

c. How likely is it that a randomly selected 2012 Prius gets less than 48MPG?

Find: $P(X < 48)$

```
pnorm(48, 53.3, 5.2)
```

```
## [1] 0.1540467
```

```
mean(mpg < 48)
```

```
## [1] 0.1573
```

d. A car should be checked out for problems if is getting an MPG in the bottom 5% of all cars like it. At what MPG should we start to worry?

Find t such that $P(X < t) = .05$

```
qnorm(.05, 53.3, 5.2)
```

```
## [1] 44.74676
```